3.6/9

$$x^3 + xy - 2x = 1$$

a) differentiate implicitly to find $\frac{dy}{dx}$

$$3x^2 + \left(\frac{d}{dx}(x)\cdot y + x\cdot\frac{d}{dx}(y)\right) - 2 = 0$$

$$3x^2 + \left(y + x\frac{dy}{dx}\right) - 2 = 0$$

$$x\frac{dy}{dx} = 2 - 3x^2 - y$$

$$\frac{dy}{dx} = \frac{2-3x^2-y}{x}$$

b) $x^3 + xy - 2x = 1$

$$xy = 1 + 2x - x^3 \qquad x^{-1}$$

$$y = \frac{1+2x-x^3}{x} = \frac{1}{x} + 2 - x^2$$

$$y' = \frac{(2-3x^2)x - (1+2x-x^3)(1)}{(x^2)} = -\frac{1}{x^2} - 2x$$

$$= \frac{2x - 3x^3 - 1 - 2x + x^3}{x^2} = \frac{-2x^3 - 1}{x^2} = -2x - \frac{1}{x^2}$$

c) $\frac{dy}{dx} = \frac{2-3x^2-y}{x}$

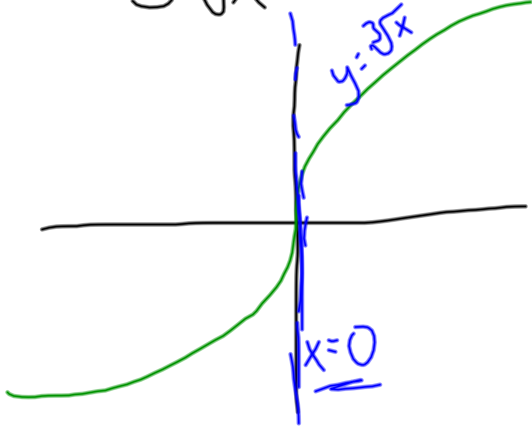
$$\frac{dy}{dx} = \frac{2 - 3x^2 - \left(\frac{1}{x} + 2 - x^2\right)}{x} = \frac{-2x^2 - \frac{1}{x}}{x}$$

$$= -2x - \frac{1}{x^2}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}} \quad (-\infty, \infty)$$

$$\frac{d}{dx}\left(x^{\frac{1}{3}}\right) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$$

all real #s
except $x=0$



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3.6|5)

$$y = x^3\left(5x^2+1\right)^{-\frac{2}{3}}$$

$$\frac{dy}{dx} = \left(3x^2\right)\left(5x^2+1\right)^{-\frac{2}{3}} + \left(x^3\right)\left(-\frac{2}{3}\left(5x^2+1\right)^{-\frac{5}{3}}\left(10x\right)\right)$$

$$= x^2\left(5x^2+1\right)^{-\frac{5}{3}}\left[3\left(5x^2+1\right) + \frac{-20}{3}x^2\right]$$

$$\frac{\left(5x^2+1\right)^{-\frac{2}{3}}}{\left(5x^2+1\right)^{-\frac{5}{3}}} = \left(5x^2+1\right)$$

$$= x^2\left(5x^2+1\right)^{-\frac{5}{3}}\left[\frac{25}{3}x^2+3\right]$$

$$= \frac{x^2\left(5x^2+1\right)^{-\frac{5}{3}}\left[25x^2+9\right]}{3}$$

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3.5/45)

$[\sec(\frac{\pi}{2}-x)]^3$

$y=\sec^3(\frac{\pi}{2}-x)$, $x=-\frac{\pi}{2}$, find eot L

Point: $(x,y)=(-\frac{\pi}{2}, \sec^3(\frac{\pi}{2}-(-\frac{\pi}{2})))$

$(-1)^3=-1$

Slope: $y'|_{x=-\frac{\pi}{2}}$

$y'=3\sec^2(\frac{\pi}{2}-x)(\sec(\frac{\pi}{2}-x)\tan(\frac{\pi}{2}-x))(-1)$

WTH is $\sec(\frac{\pi}{2}-x)=\frac{1}{\cos(\frac{\pi}{2}-x)}$

when $x=-\frac{\pi}{2}$ $\Rightarrow \frac{1}{\cos(\pi)}=\frac{1}{-1}=-1$

what is $\tan\pi=0$

$\therefore y'(x=-\frac{\pi}{2})=3(-1)^2(-1)(0)(-1)=0$

$y-(-1)=0(x-(-\frac{\pi}{2}))$

The equation of a line
with slope m
through the point (x_0, y_0)
is:

$$y - y_0 = m(x - x_0)$$

this follows from

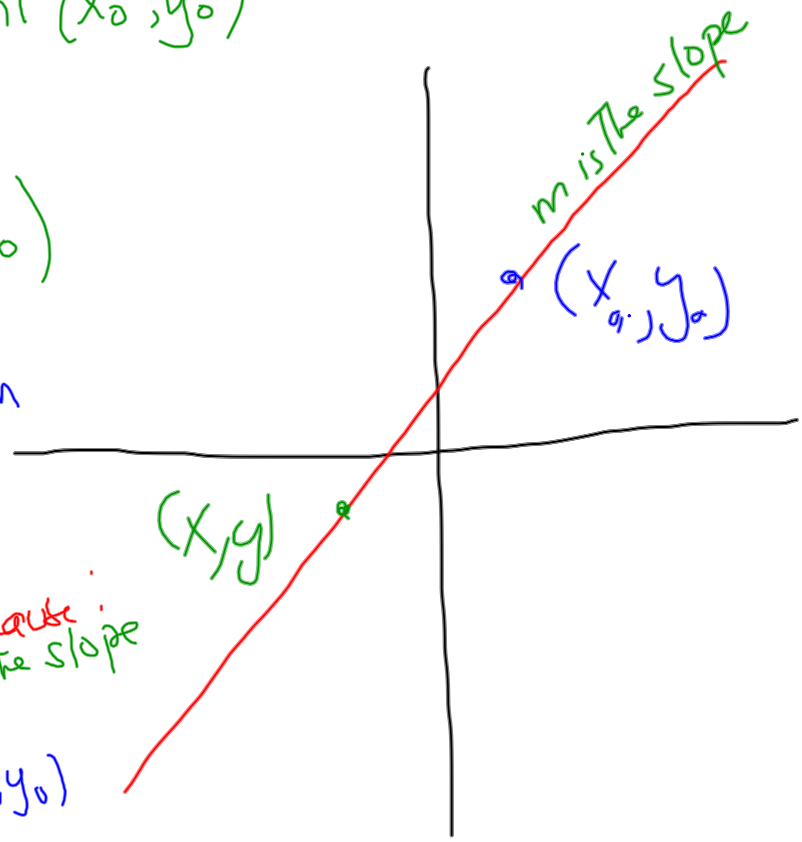
$$m = \frac{y - y_0}{x - x_0}$$

this follows because:

m is the slope

(x_0, y_0)

(x, y)



3.5/47)

$y = \tan(4x^2),\ x = \sqrt{\pi}$ et1

Point: $(x, y) \Rightarrow \left(\sqrt{\pi}, \tan\left(4(\sqrt{\pi})^2\right)\right)$

$= \left(\sqrt{\pi}, \tan(4\pi)\right) = \left(\sqrt{\pi}, 0\right)$

SLOPE

$y' = \sec^2(4x^2)\,(8x)$

$y'\Big|_{x=\sqrt{\pi}} = 8\sqrt{\pi}\sec^2(4\pi) = 8\sqrt{\pi}(1)^2 = 8\sqrt{\pi}$

$\sec(4\pi) = \dfrac{1}{\cos(4\pi)} = \dfrac{1}{1} = 1$

$y - 0 = 8\sqrt{\pi}\left(x - \sqrt{\pi}\right)$

$y = 8\sqrt{\pi}\,x - 8\pi$

3.6/2

$$y = \sqrt[3]{2+\tan(x^2)} = (2+\tan(x^2))^{\frac{1}{3}}$$

$$\frac{dy}{dx} = y' = \frac{1}{3}\left(2+\tan(x^2)\right)^{-\frac{2}{3}}\left(\sec^2(x^2)(2x)\right)$$

$$= \frac{2x\sec^2 x^2}{3}(2+\tan(x^2))^{-\frac{2}{3}}$$

$$= \frac{2x}{3\cos^2(x^2)}\left(2+\tan x^2\right)^{-\frac{2}{3}}$$

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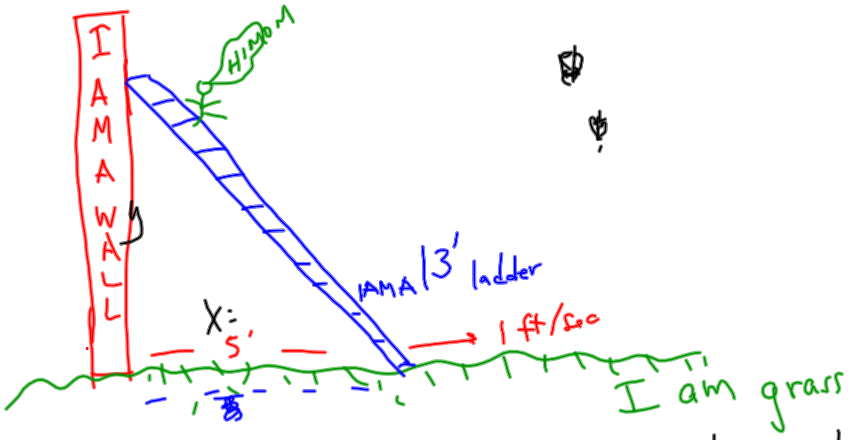
3.7 related rates



1) find an equation in real life that is always true.

$$x^2+y^2=13^2$$

Now: think about x, y as if they were x(t), y(t)

2) differentiate implicitly with respect to t.

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0 \Rightarrow x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

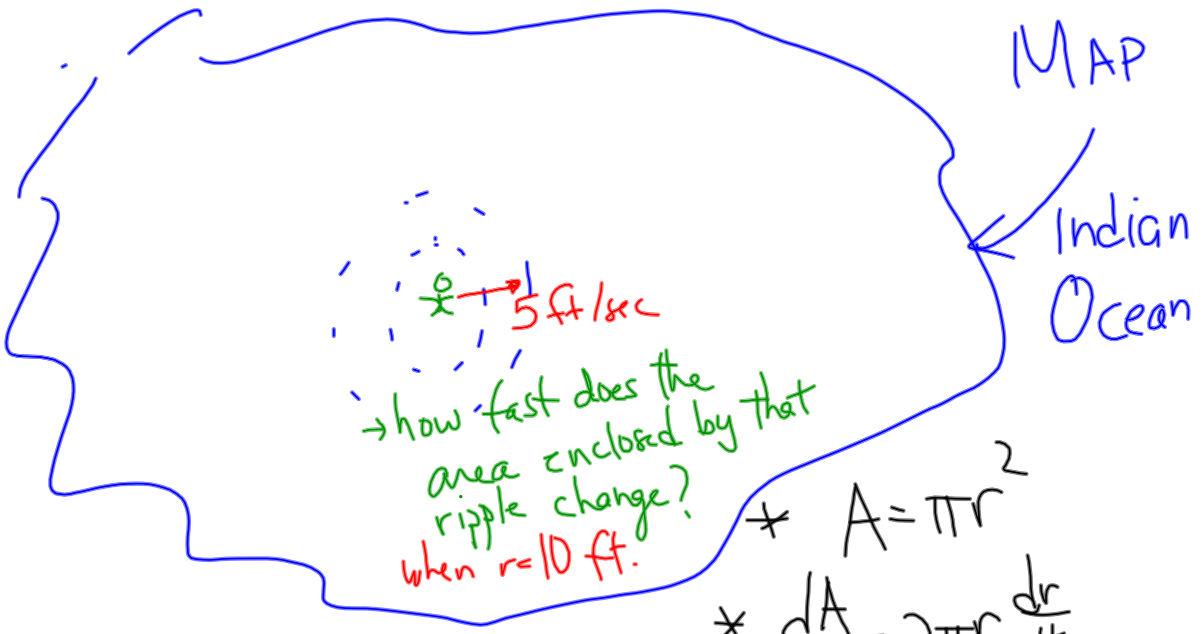
this is a RELATIONSHIP between $x, y, \frac{dx}{dt}, \frac{dy}{dt}$ ... It is ALWAYS true.

3) Now substitute in values that I am interested in.

$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$$5(1)+\left(\sqrt{13^2-5^2}\right)\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=\frac{-5}{12}\text{ ft/sec}$$



→ how fast does the area enclosed by that ripple change? when $r = 10$ ft.

* $A = \pi r^2$

* $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

* $\frac{dA}{dt} = 2\pi(10)(5)$

$= 100\pi \frac{ft^2}{sec}$

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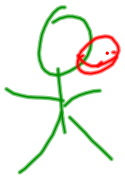

bin is blowing 100 cm^3 of air into the bubble every sec.

how is the radius of the bubble changing after 5 sec?

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100 = 4\pi\left(\sqrt[3]{\frac{375}{\pi}}\right)^2 \frac{dr}{dt}$$

$$\frac{25}{\pi}\,\frac{\pi^{2/3}}{375^{2/3}}\ \text{cm/sec} = \frac{dr}{dt}$$

After 5 sec

$$5(100) = \frac{4}{3}\pi r^3$$

$$\frac{375}{\pi} = r^3$$

$$\sqrt[3]{\frac{375}{\pi}} = r$$

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